T R Anantharaman –Education and Research Foundation
www.tra-erf.org

# Elastic modulus

## Basics and significance

P Rama Rao and V Chandrasekaran
ARCI, Hyderabad

**We claim no originality for the material presented. We have compiled such technical information as we considered useful in our effort to communicate to you the basics, significance and applications of elasticity.**

**Recognizing the importance of quantitative measurement in order to have a feel for the stiffness of a given material, we will demonstrate one known method of measurement of Young's modulus of steel.**

**I was like a boy playing on the seashore, and diverting myself now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.**

***Sir Isaac Newton, Letter to Robert Hooke, February 5, 1675***

**Professor TR Anantharaman (1925 – 2009) was an exceptional teacher.**

**He was the research guide of one of the speakers (P Rama Rao).**

**He built up the country’s leading research school in Materials science and Technology at Banaras Hindu University during 1962 – 1982 with absolutely meager funds.**

**This lecture draws inspiration from him.**

# Outline

- **Definitions and stress-strain curve**
- **Atomic basis of elasticity**
- **Stiffness and Young’s modulus of elasticity**
- **Magnitude of elastic modulus in different materials**
- **Extremities in elasticity observed in new materials:**

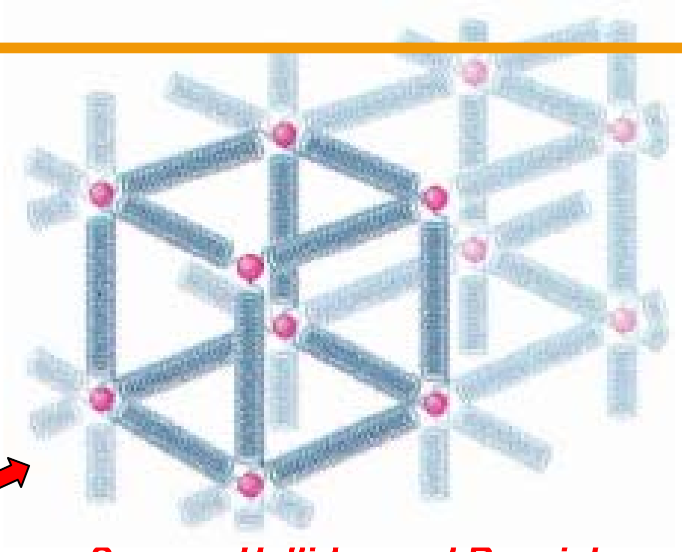
  **(A) graphene, (B) gum metal**
- **Computational approach to elasticity**

# Definitions and stress – strain curve

# Elasticity

**The atoms of a solid are distributed on a repetitive three dimensional lattice.**
**The springs represent inter-atomic forces.**

**The lattice is remarkably rigid, interatomic springs are extremely stiff.**



***Source: Halliday and Resnick Fundamentals of physics, Jearl Walker, 9th edition John Wiley & Sons, Inc, 2011***

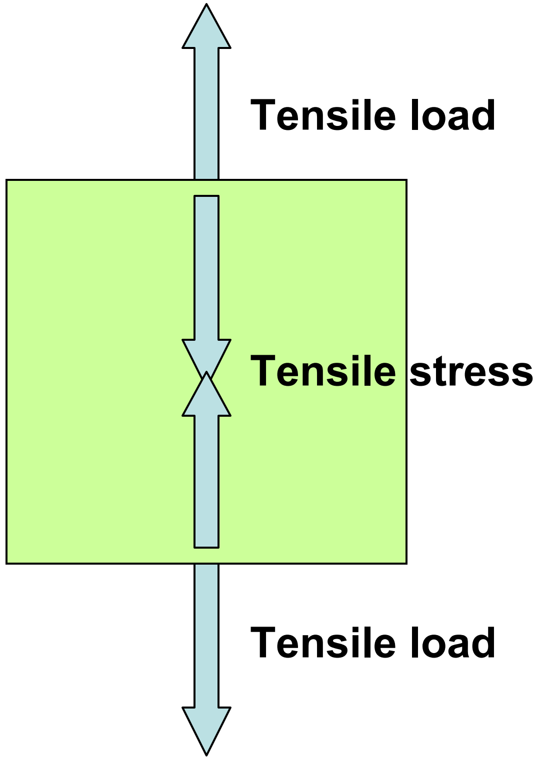
**By application of suitable forces, the rigid bodies can be made to change both in size and shape, i.e. their dimensions can be changed slightly by pulling, pushing, twisting, or compressing.**

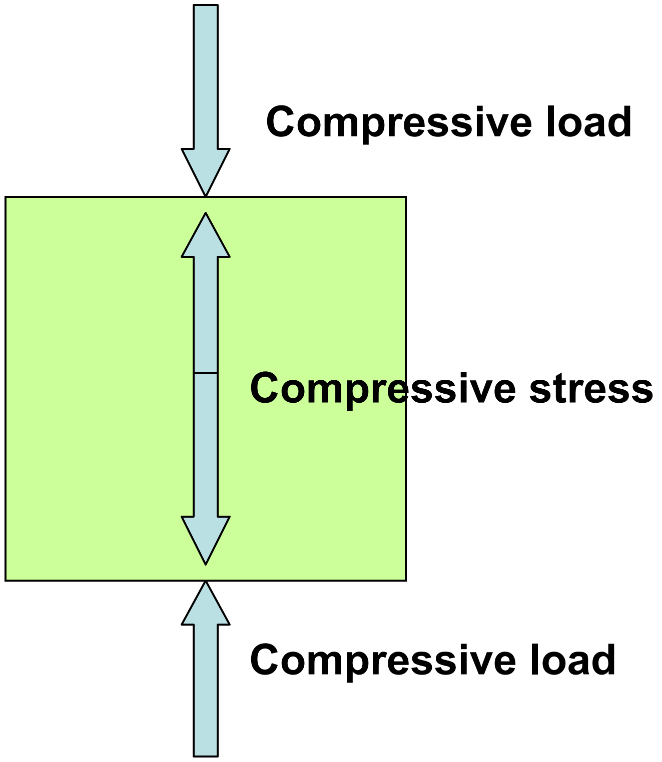
**When the dimensional changes are not too great, the solid returns to the original shape and size, after the deforming forces have ceased to act.**
**This property of solids is termed "elasticity" and is common to all solids.**

# Stress

When a body is stressed, forces of reaction come into play internally in it, resisting further deformation and tending to restore the body to its original condition.

The restoring or recovering force per unit area set up inside the body is called “STRESS”



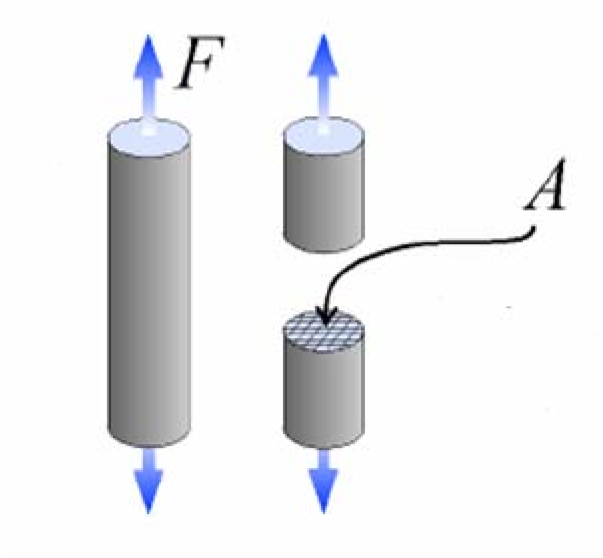



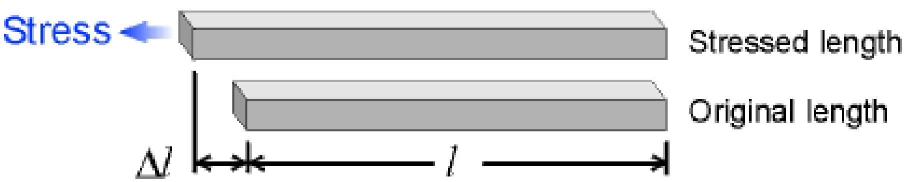

**Stress is a measure of the internal resistance in a material to an externally applied load.**

For direct compressive or tensile loading, the stress is defined as

$$\text{Stress, } \sigma = \frac{\text{Deforming force (F)}}{\text{Area of cross section (A)}}$$

**Strain** is a measure of the deformation produced by the influence of stress.




$$\text{Strain, } \varepsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

*Source of the figures : Internet*

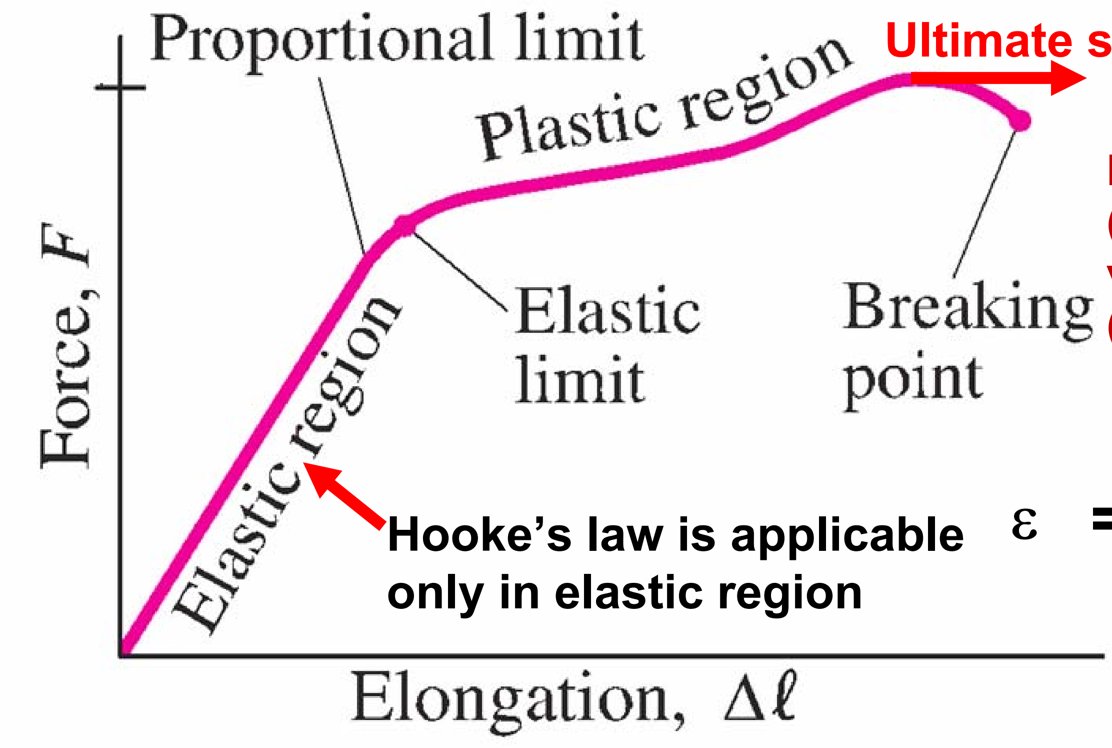
Strain is dimensionless and represents unit deformation

**One pascal (SI unit of stress) = one newton of force per square meter of area (1 N/m2).**

# Applied force vs elongation for a typical metal under tension

Stress – Strain curve

Elastic modulus can be in principle, obtained from the slope of the elastic region. However, this is not accurate, because of measurement errors.




Better ways:
(1)By measuring natural frequency of vibration
(2)By measuring velocity of sound
$C = [E/\rho]^{1/2}$ where $\rho$ is the density

$$\varepsilon = \frac{\sigma}{E}$$

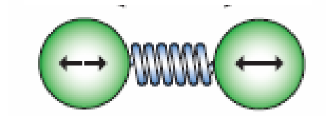
*Source of the figure : Internet*

**The ultimate strength is the maximum stress attained prior to failure. Modulus is a measure of the stiffness of the material.**

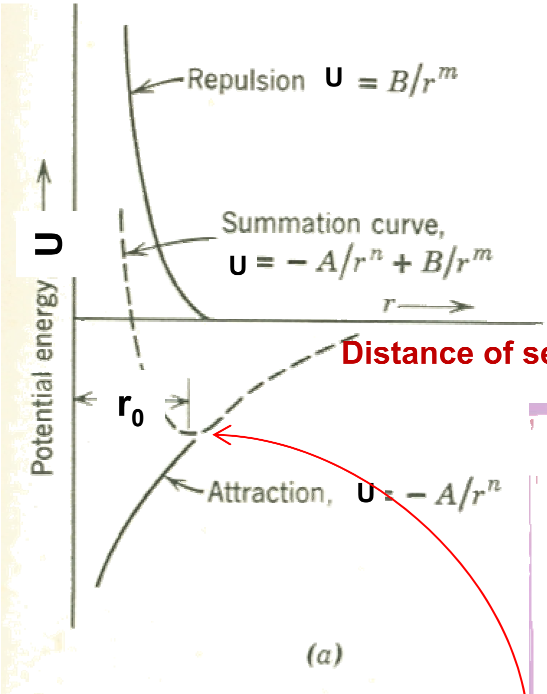
# Atomic basis of elasticity

# Atomic basis of elastic behavior, Condon-Morse Curves

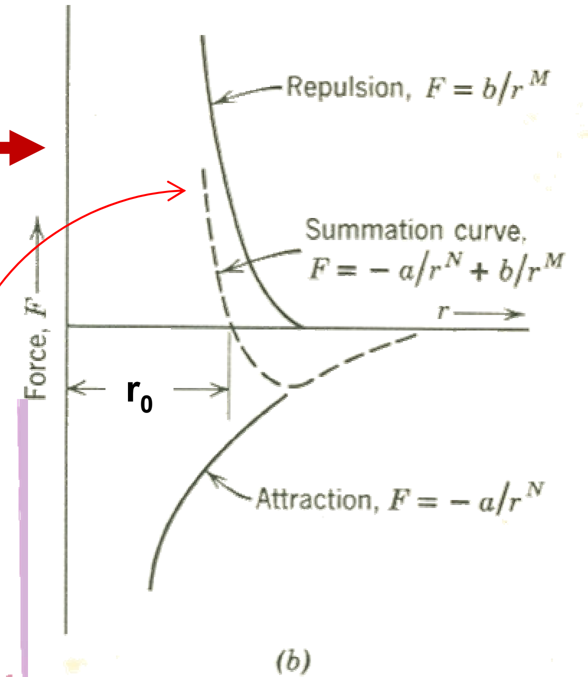
Inter-atomic separation, r

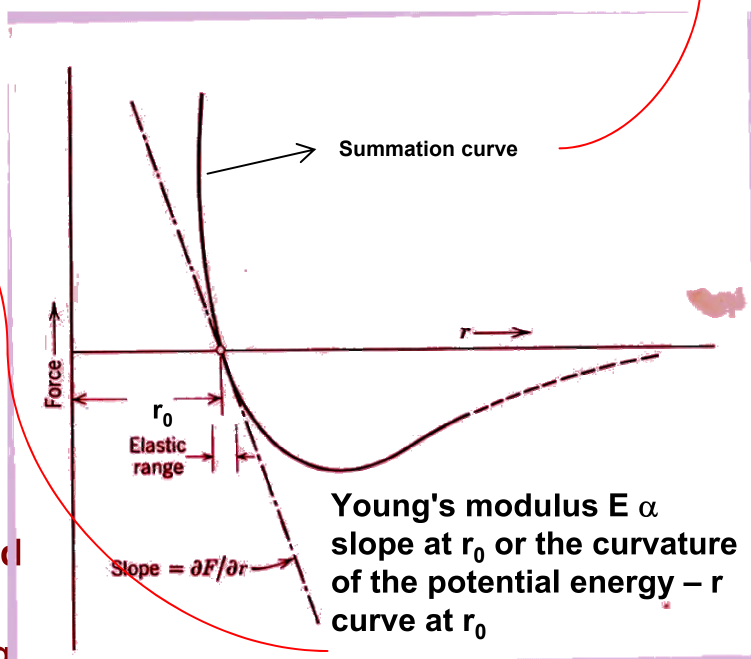


**Variation of potential energy, U, with distance of separation ,r, between a pair of atoms**




**Differentiating U and simplifying, the net force of attraction and repulsion is given by**




Young's modulus E α slope at $r_0$ or the curvature of the potential energy – r curve at $r_0$

$$V = \frac{-A}{r^n} + \frac{B}{r^m}$$

$$F = \frac{-a}{r^N} + \frac{b}{r^M}$$

**A and B : proportionality constants for attraction and repulsion**

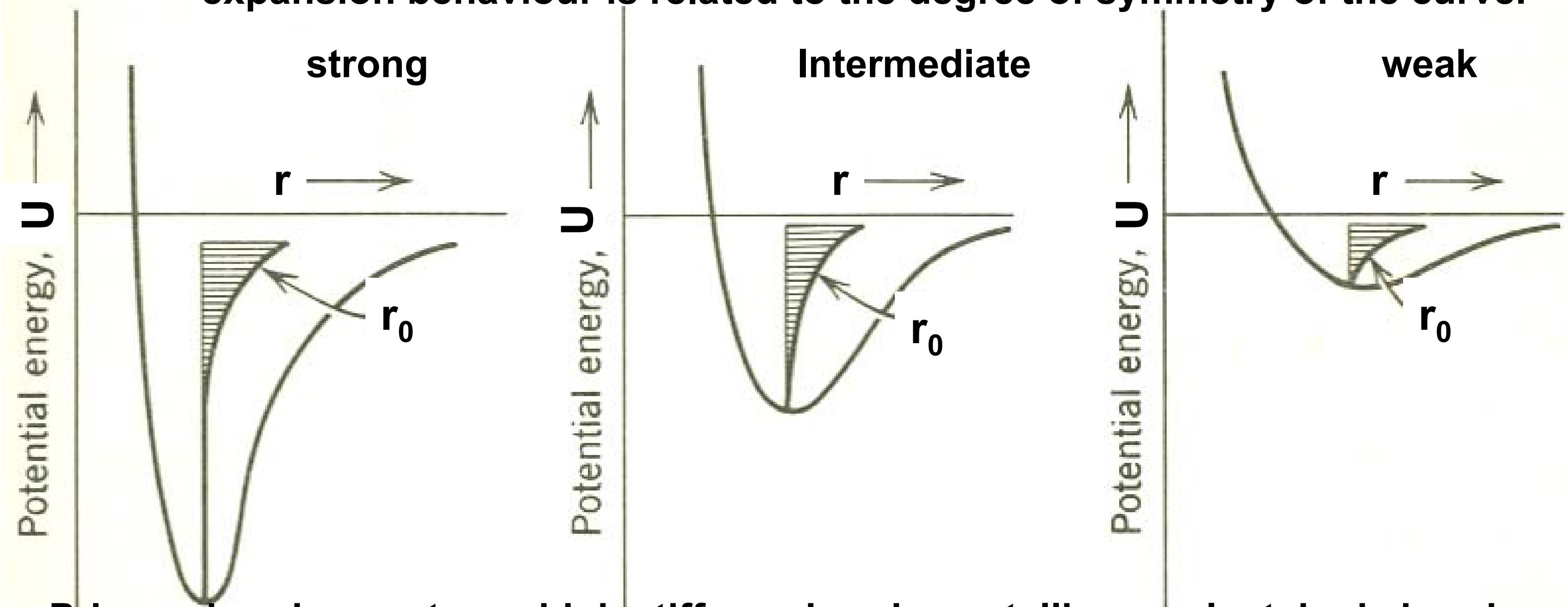
**n and m : exponents giving the appropriate variation of U with r**

**At the equilibrium spacing,$r_0$, U is minimum and net F = 0**

*Source: The structure and properties of materials, Vol III, HW Hayden, William G Moffatt and John Wulff , John Wiley & sons, Inc.1966*

# Melting point, sublimation, Young's modulus and coefficient of thermal expansion are qualitatively related to Condon-Morse curves

**The sublimation temperature is directly related to the depth of the trough; Young's modulus is inversely related to the radius of curvature at the bottom of the trough; and thermal expansion behaviour is related to the degree of symmetry of the curve.**




**Primary bonds are strong high stiffness bonds: metallic, covalent, ionic bonds.**
**Melting point: 1000 – 4000 K.**
**Secondary bonds are weak low stiffness bonds: Van der waals and hydrogen bonds.**
**Melting point: 100 – 500 K.**

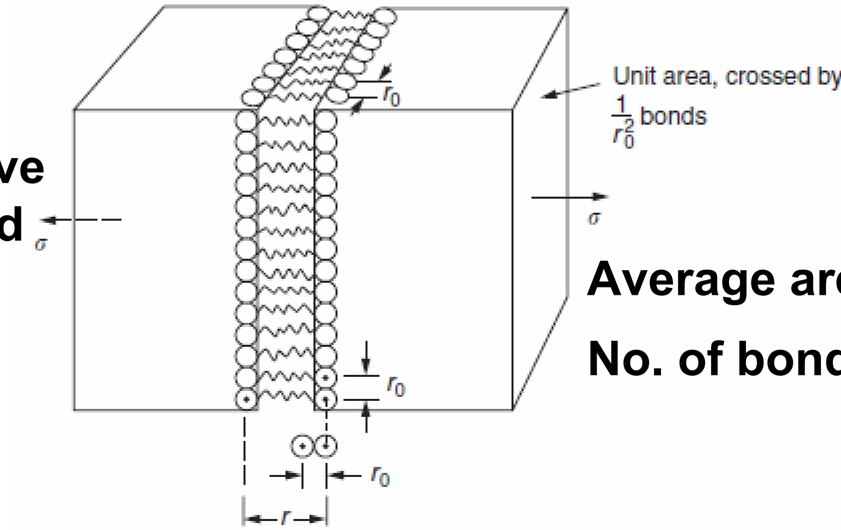
**Besides the strength of the interatomic bonds, the elastic modulus is dependent on ways in which atoms are packed together – which is different in different types of materials.**

*Sources: (1) The structure and properties of materials, Vol III, HW Hayden, William G Moffatt and John Wulff , John Wiley & sons, Inc.1966*
*(2) Engineering materials-1*
*Michael Ashby and David RH Jones, Elsevier Butterworth – Heinemann , 2005*

# Stiffness and Young's modulus of a material

# Young's modulus of a crystal from bond stiffness

Atomic bonds behave like little springs and Spring stiffness = S




Average area per atom (bond) : $r_0^2$

No. of bonds per unit area, $N = 1/r_0^2$

The force between a pair of atoms, stretched apart to a distance r is

$$F = S_o ( r- r_o )$$

The stress ($\sigma$ ) is the total force exerted across unit area and is

$$\sigma = N S ( r- r_o) = \{S (r- r_o) / r_o^{2} \}$$

The strain is given by $\varepsilon = (r - r_o ) / r_o$

(ratio of displacement $(r - r_o)$ to initial spacing $r_o$)

$\therefore$ Young's modulus is $E = (\sigma / \varepsilon) = (S_o / r_0 )$

*Source : 1) Engineering materials-1*
*Michael Ashby and David RH Jones, Elsevier Butterworth – Heinemann , 2005*
*2) Materials Engg,Science, Processing and Design by Michael Ashby et al Butterworth-Heinemann, 2007*

**$S_o$ can be theoretically calculated from U( r ) { Condon – Morse curves ) and E can be estimated**

# Magnitude of elastic modulus in different materials

## The menu of engineering materials

**Materials → Backbone to our civilization**

### Common examples of each material family

**Metals**
Steels,
Cu alloys

**Polymers**
Polyesters,
Epoxies

**Elastomers**
Butyl rubber,
silicones

**Hybrids**
Composites,
Foams

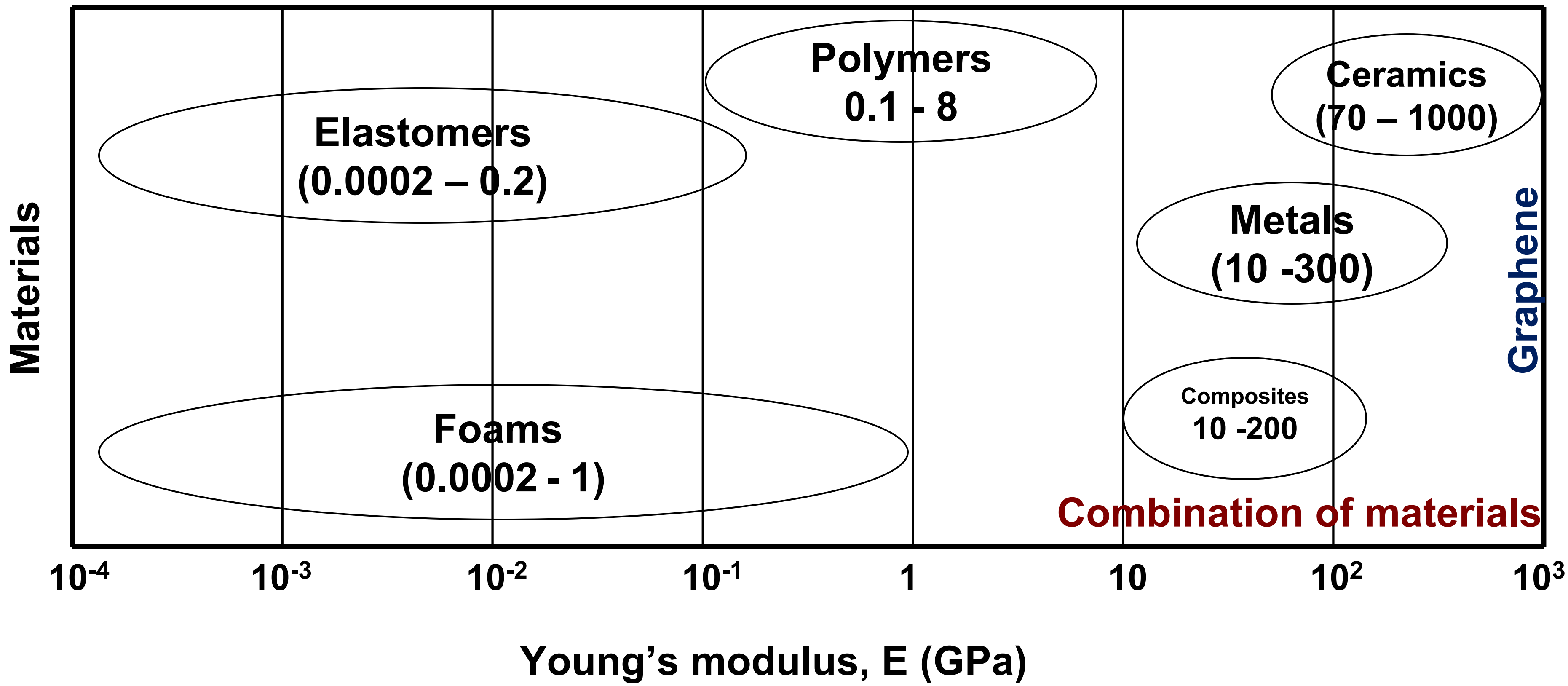


**Glasses**
Silica glass,
Glass ceramics

**Ceramics**
Aluminas,
Silicon carbides

*Source: Materials Engg,Science, Processing and Design by Michael Ashby et al Butterworth-Heinemann, 2007 and Internet*

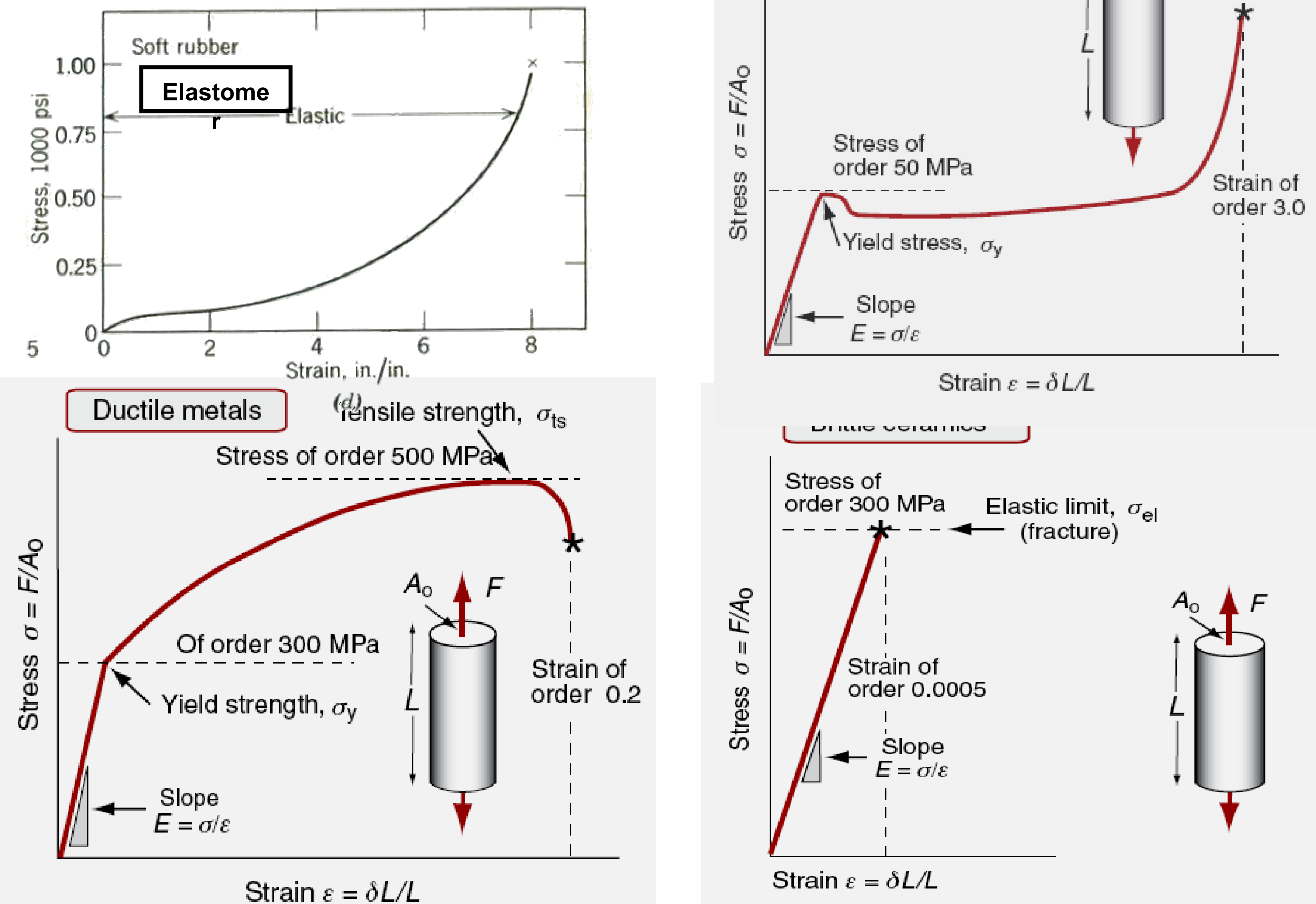
# Young's modulus of different materials




*Source : Materials Engg,Science, Processing and Design by Michael Ashby et al Butterworth-Heinemann, 2007*

**Material families have discrete range of Young's modulus**

# Tensile stress-strain curves for polymers, metals and ceramics.

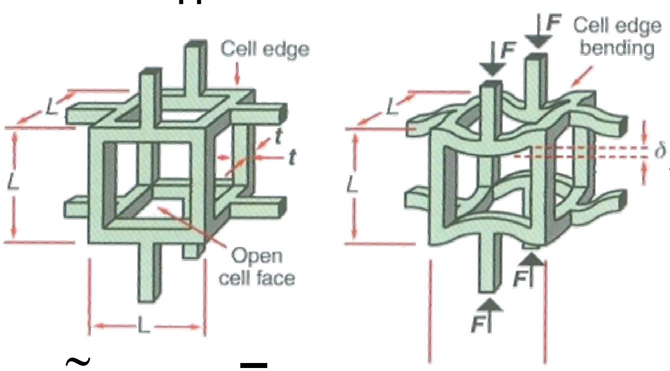



*Source: 1) Materials Engg,Science, Processing and Design by Michael Ashby et al Butterworth-Heinemann, 2007*
*2) The structure and properties of materials, Vol III, HW Hayden, William G Moffatt and John Wulff , John Wiley & sons, Inc.1966*

# Atomic weights

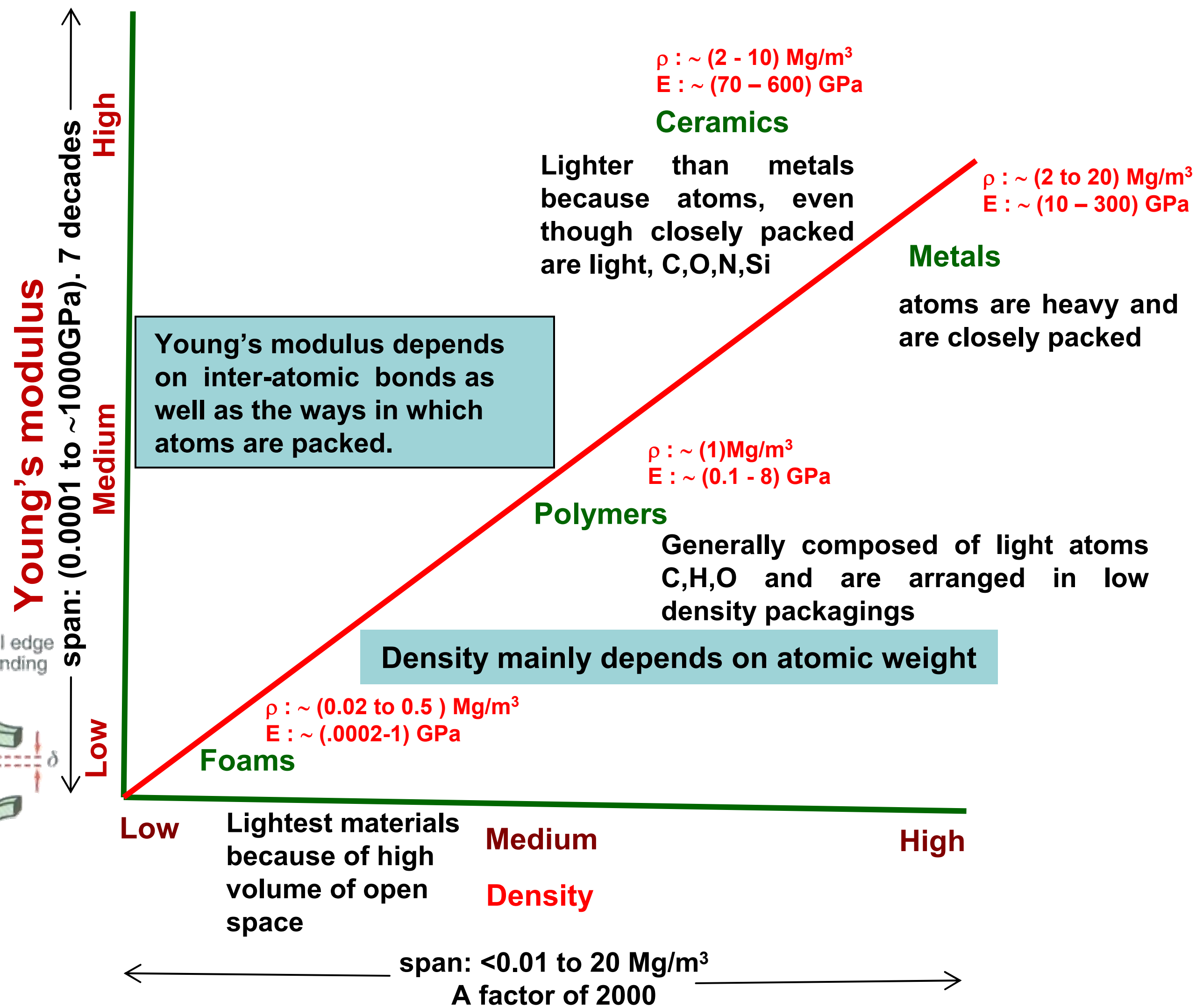
**H:1, C:12, N:14, O:16**
**Si:28, Fe:56, U:238**

**CORK**
**(Foam structure**
**Poisson's ratio=0**
**use: stopper for wine bottles**




$$\frac{\tilde{E}\ (\text{Foam})}{E\ \text{Solid}} = \left(\frac{\bar{\rho}\ (\text{Foam})}{\rho\ \text{solid}}\right)^2$$

**So $\tilde{E}$ can be manipulated by changing ρ**



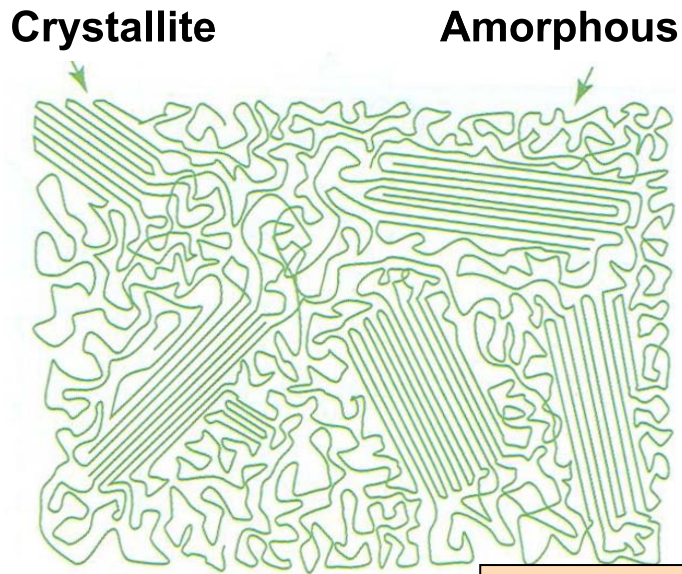

*Source: Materials Engg,Science, Processing and Design by Michael Ashby et al Butterworth-Heinemann, 2007*

# Bond stiffness and Young's modulus for some bond types

| Bond type | Examples | Bond stiffness, S (N/m) | Young's modulus, E (GPa) |
| --- | --- | --- | --- |
| Van der Waals (e.g. Polymers) | Waxes | 0.5 - 1 | 1 - 4 |
| Hydrogen bond (e.g. $H_2O$ –$H_2O$) | Polyethylene | 3-6 | 2 – 12 |
| Ionic (e.g. Na-Cl) | Sodium chloride | 8-24 | 32 – 96 |
| Metallic (e.g. Cu-Cu) | All metals | 15-75 | 60 – 300 |
| Covalent (e.g. C-C) | Carbon-carbon bond | 50-180 | 200 - 1000 |

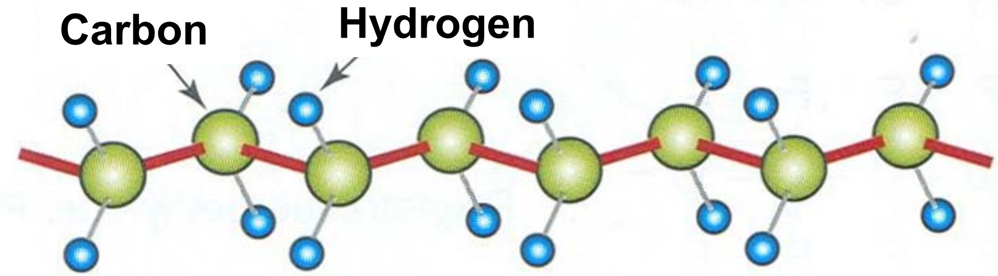
**Source: Materials Engg,Science, Processing and Design by Michael Ashby et al** Butterworth-Heinemann, 2007

# Influence of structure (C-C bond) on Young's modulus

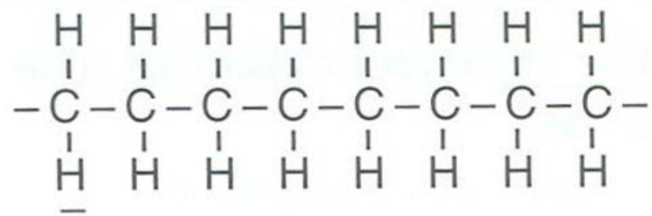



bulk polyethylene

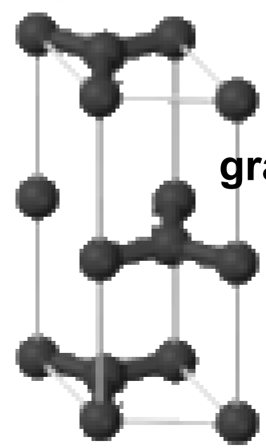
E = 0.2 GPa




cold drawn polythylene



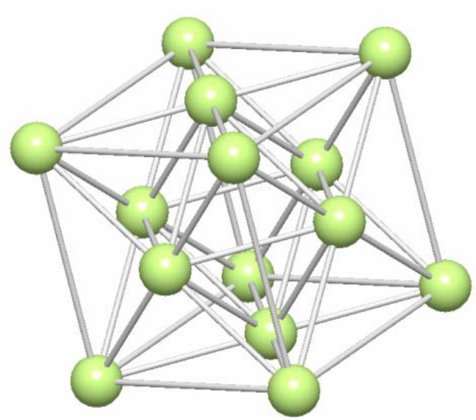
E = 300 GPa



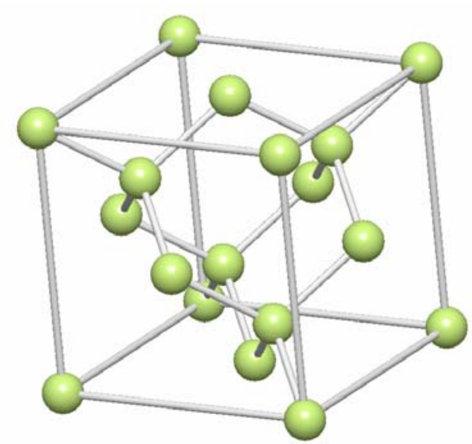
graphite

E = 1020 GPa

E = > 1000 GPa

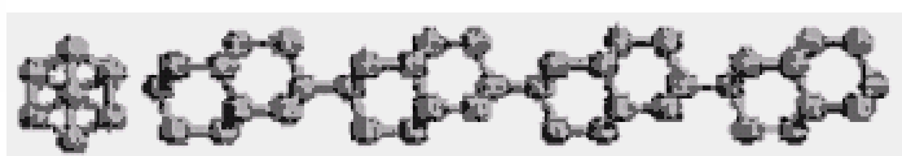


fcc diamond (Theoretical)



dc diamond

E = 1050-1200 GPa



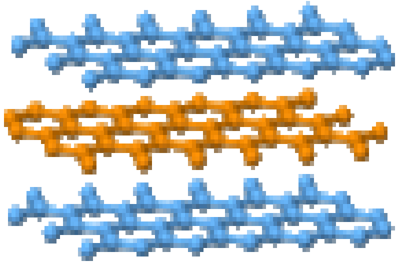
diamond nano-rods

E = 1214 GPa

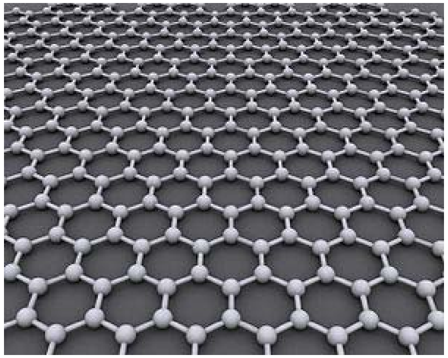
**Extremities in elasticity observed in new materials: (A) graphene, (B) gum metal**

# Graphene:
# Nobel prize in Physics 2010

**Graphite**
Consists of many layers of six-membered carbon rings

**Graphene**
is an isolated atomic plane of graphite with atoms arranged in a regular hexagonal pattern.



Andre Geim

Schotch tape method in which an adhesive tape is used to peel off a single layer-which is graphene – from a large crystal of pure graphite.

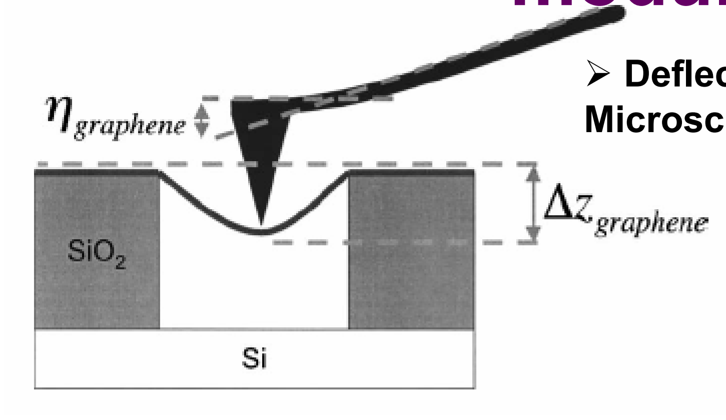
Konstantin Novoselov

*Source of figures : Internet*

**Atoms on a small scale behave like nothing on a large scale.**

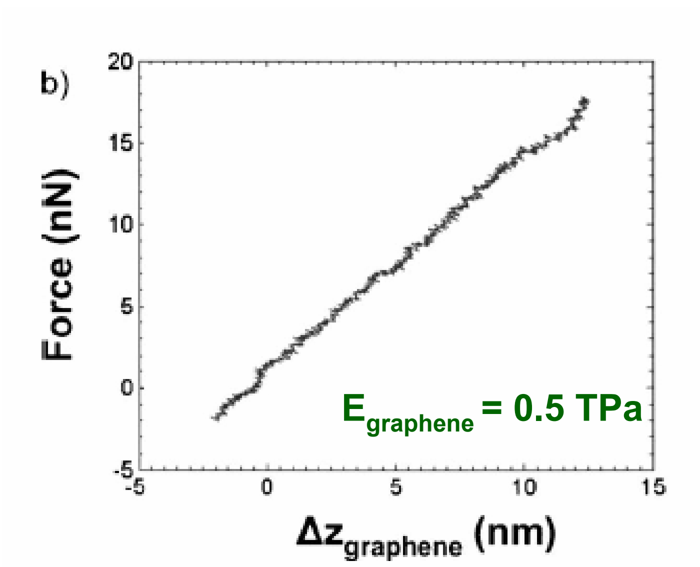
**As we go down and fiddle around with atoms down more, we are working with different laws.** (Richard Feynman)

# Pin and deflection method to measure the Young's modulus of graphene.




➢ Deflection of the graphene sheet is measured using Atomic Force Microscope.

Shows an AFM tip that is deflected while pushing down on a suspended graphene sheet. From the deflection, the force constant is calculated from which Young's modulus is evaluated.




## Attributes

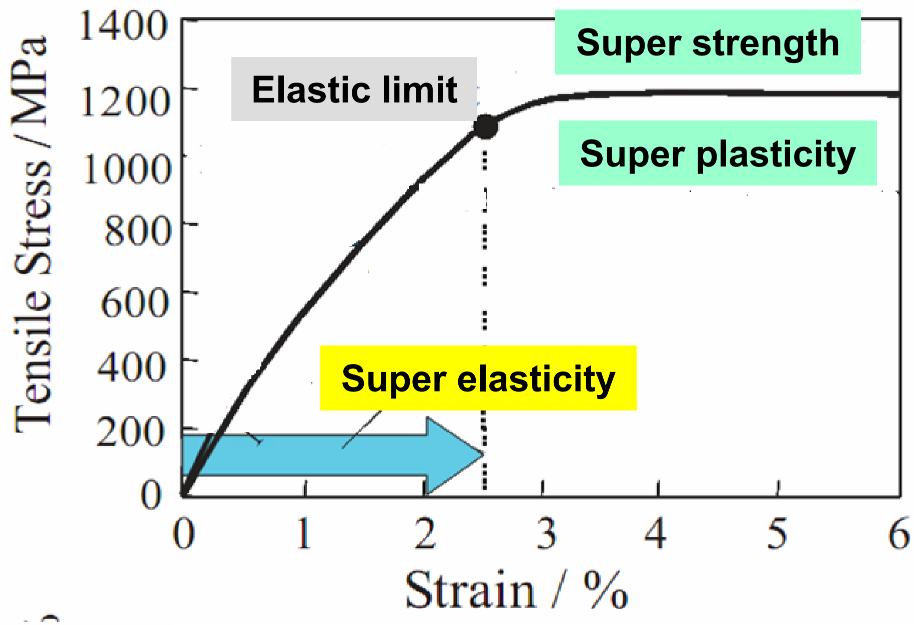
Not only graphene is lighter, stronger, harder and more flexible than steel, it is also a recyclable and sustainably manufacturable product that is eco-friendly and cost effective in its use.

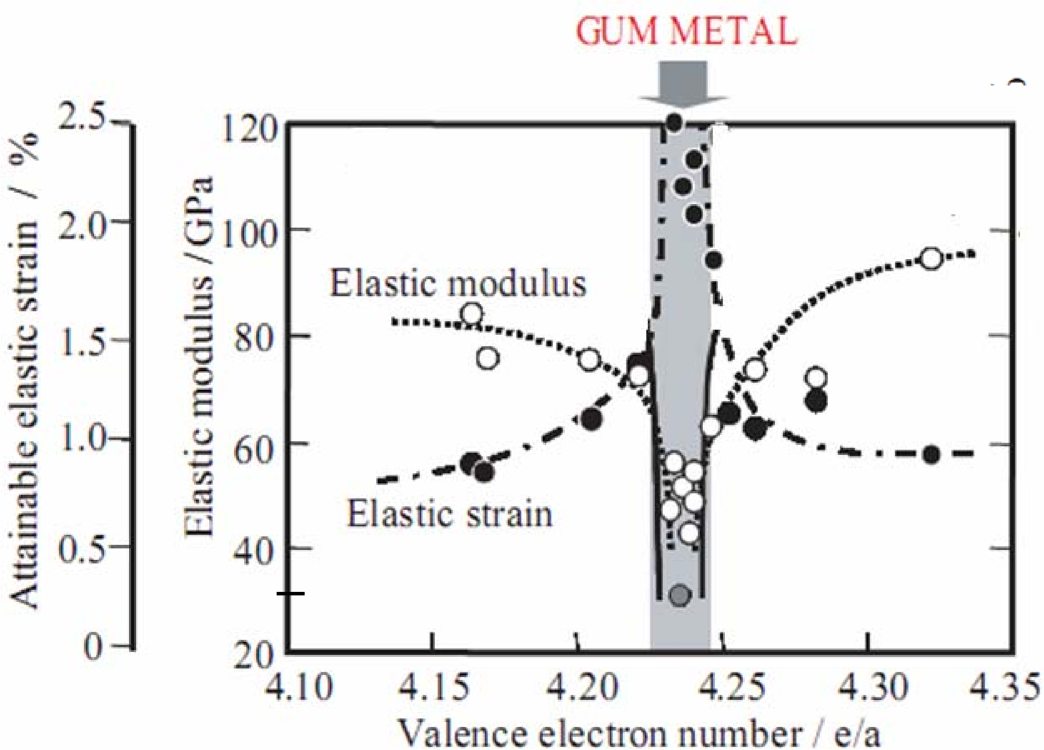
*Source: IW Frank et al*
*J. Vac. Sci. Technol. B 25(6) Nov/Dec 2007*

Source: Wikipedia

# Gum metal

**Ti-23Nb-0.7Ta-2Zr-1.2 O alloy –an inter- metallic {Ti3(Ta+Nb+V)(Zr,Hf) +O } with bcc structure**




**Anomaly in properties of Ti-Nb-Ta-Zr-O alloys at e/a ratio of 4.23- 4.24**

*Source: Saito et al., Science, 2003 (300) pp 464 and Kazuaki Nishimo, R&D Review of Toyoto CRDL, Vol 38 (3) 50*

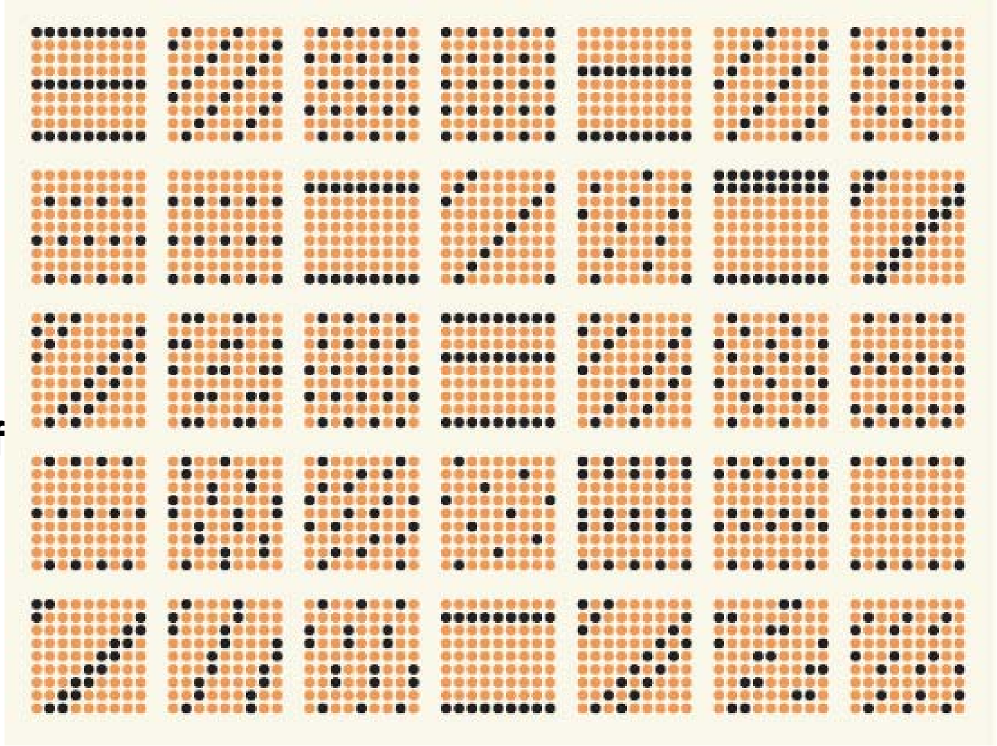
# Computational approach to elasticity

❖ Thermodynamic and elastic properties of alloys can be predicted through computational approach.

❖ Maisal et al ( Nature, 491, Nov. 2012) developed a new quantum mechanics based computational methodology known as "cluster expansion" to calculate the properties of substitutional alloys.

❖ The approach facilitates to understand:

- ❖ correlation between the thermodynamic and elastic properties.
- ❖ Tuning the elastic stiffness of alloys using materials substitution.
- ❖ rapid prediction of properties



Several atomic configurations of possible alloys having a square lattice.
Atoms of main metal : orange
Substitutional atoms : black

# Thank you